DATA PROCESSING AND PLANNING OF NONSTATIONARY THERMOPHYSICAL EXPERIMENTS

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Results are represented of an experimental computational determination of the temperature dependence of the effective heat conduction coefficient of a glass plastic and analysis is given of the reliability of the data obtained by using computational experiments and planning of the temperature measurements.

Carefully analyzing and giving a foundation to the reliability of the results being obtained is necessary when conducting experimental—computational thermophysical investigations based on application of the inverse problem method to process the experimental information. To this end a combination of computational experiments simulating the different conditions for performing the investigations (see [1], say) and the optimal planning of the temperature measurements can be used wherein the quality of the measuring scheme [2] realized in the experiments can be exposed. Application of the approach mentioned is examined in this paper in an analysis of the data of nonstationary thermophysical experiments conducted to determine the temperature dependence of the effective heat conduction coefficient of a glass-plastic on a silicon organic binder from the solution of the inverse problem.

The tests were performed in a wind tunnel with the following gas flow parameters: $P_0 = 5 \cdot 10^5$ Pa, $T_0 = 3590$-4340 K, M = 6.5. The model in the form of a slab with the specimen being investigated arranged therein was placed in the working chamber at a 20° angle to the incoming flow. Thermocouples VR5/20 of 0.2-mm diameter, soldered at the butt, were mounted in the specimens executed in the shape of 20-mm-thick rectangular plates of 128 × 170 mm at a different depth from the surface being heated. The microcomputer "Élektronika DZ-28" and the automatic potentiometer KSP-4 were used to process the primary information from the thermocouples. The error in measuring the temperature was 1.5-2% at interior points of the specimens.

Experimental data obtained for the two specimens are analyzed below. Six thermocouples were mounted in the first at the following distances from the surface being heated: $d_1 = 2.8$ mm; $d_2 = 5.6$ mm; $d_3 = 8.4$ mm; $d_4 = 11.2$ mm; $d_5 = 17.2$ mm; and $d_6 = 20$ mm. Four thermocouples were in the second specimen at $d_1 = 2.8$ mm; $d_2 = 8.4$ mm; $d_3 = 11.2$ mm; and $d_4 = 20.5$ mm. The thermocouple location was determined by using x-ray diffraction. The diagram of the thermal sensor location in the specimens is displayed in Fig. 1. The thermal diagrams of the experiments are presented in Fig. 2. The known dependence of the bulk specific heat of a material on temperature C(T) is represented in Fig. 3a.

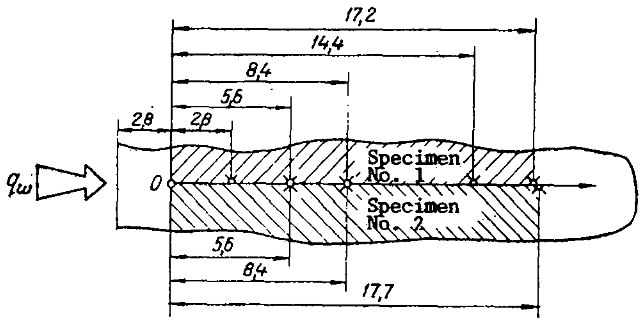

Fig. 1. Diagram of the thermal sensor arrangement in the specimens.

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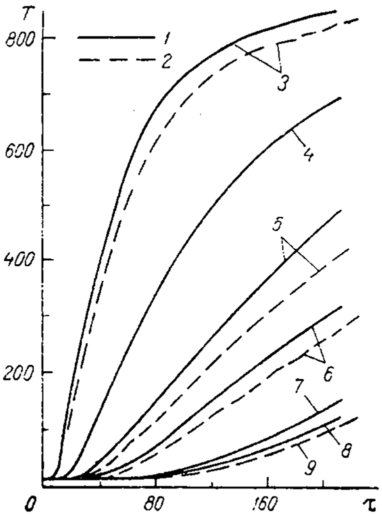

Fig. 2. Results of measuring the specimen temperature: 1) specimen No. 1; 2) specimen No. 2; 3) X = 0 mm; 4) 2.8; 5) 5.6; 6) 8.4; 7) 14.4; 8) 17.2; 9) 17.7. T, C; τ, sec.

The inverse problem was solved by using the algorithm proposed in [3] when processing the experimental data to determine the temperature dependence of the heat conduction coefficient. The desired function λ(T) was here represented in the form of a cubic B-spline with "natural" boundary conditions

$$\lambda(T) = \sum_{k=1}^{m} \lambda_k B_k(T), \ \lambda''(T_{min}) = \lambda''(T_{max}) = 0; \tag{1}$$

where the minimal T_{min} and maximal T_{max} values of the temperatures were determined from the given boundary conditions of the first kind shaped by readings of the first and last thermocouple. A search for the vector of the coefficients $\bar{\lambda} = \{\lambda_k\}_1^m$ in (1) was realized in solving the inverse problem by minimization by the method of conjugate gradients of the rms functional of the residual

$$J = \sum_{i=1}^{N} \int_0^{\tau_m} [T(X_i, \tau) - f_i(\tau)]^2 d\tau, \tag{2}$$

where N is the quantity of internal thermocouples, and $T(x_i, \tau)$, $f_i(\tau)$ are the computed and measured values of the temperature at the site of the i-th thermocouple installation.

The computational experiments show [1] that the accuracy of the inverse problem solution is determined to a considerable extent by the scheme used for the temperature measurements. Independence of the results of the solution from the magnitude of the initial approximation to the desired parameters λ given a priori in the iteration algorithm can be the criterion for correctness of the selection of the measuring scheme. The results of restoring the dependence λ(T) for different initial approximations are represented in Fig. 3 for both the specimens under consideration. It is seen that the results of solving the inverse problem obtained for the first specimen are independent of the magnitude of the initial approximation while this dependence is quite essential for the second specimen. It is remarkable that the functional (2) achieved approximately identical values, close to the magnitude of the integral measurement error in all cases during its minimization.

It follows from the analysis performed that reliable values of the heat conduction coefficient are obtained just in the first of the two experiments under consideration. Because of the nearness of the mutual specimen thicknesses and the laws of temperature variation at the point X = 0 over time for both experiments, such a situation is due to the presence of the thermocouple at the point X_1 = 2.8 mm in the first specimen.

A parametric analysis of the accuracy of the solution of the inverse problem under consideration was performed to confirm the results and deductions obtained by the method of a computational experiment. The quantity of internal thermal sensors and their arrangement in the specimen under investigation was variated here.

The simulation was realized in the following sequence. Boundary conditions of the first kind from experiment No. 1 were given on the specimen boundaries. Then by using the thermophysical characteristics (Fig. 3a) the direct problem of heat conduction was solved numeri-

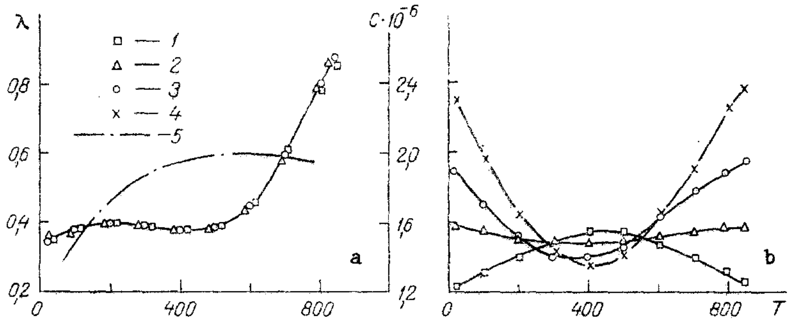

Fig. 3. Dependences λ(T) obtained during processing of experimental data for specimens No. 1 (a) and No. 2 (b) and assignment of different initial approximations $\lambda^{(0)}$: 1) $\lambda^{(0)} = 0.2$ W/(m·deg); 2) 0.4; 3) 0.6; 4) 0.8; dependence C(T). C, J/(m³·deg); T, °C.

cally and the time-varying values of the temperature was computed at several interior points of the specimen. These values were utilized as initial data for solving for the solution of the model inverse problem to restore the temperature dependence of the heat conduction coefficient not known at this stage of the computations. Single-valued solvability of the inverse problem being analyzed can be achieved when performing measurements at one interior point of the specimen [5]. Starting from this, it was assumed $N \geq 1$. The methodological error of the inverse problem solution was estimated from the formula

$$\varepsilon_\lambda = (\max|\lambda(T) - \lambda^{\text{exact}}(T)|)/\lambda^{\text{exact}}_{\max}(T), \quad T \in [T_{\min}, T_{\max}], \tag{3}$$

where λ(T) and $\lambda^{\text{exact}}(T)$ are the restored and the "exact" values of the heat conduction coefficients.

The dependence of ε_λ on the installation coordinates for one sensor is shown in Fig. 4. It is seen that high accuracy of the inverse problem solution with a methodological error less than 1% is assured only in the case of thermal sensor installation in a sufficiently narrow domain extending ~4 mm near the surface being heated.

Simulation taking account of the redundant "experimental" information was performed for two thermal sensors. Variation of the installation coordinates of the second sensor was realized for different fixed locations of the first. The errors in restoring the heat conduction coefficient obtained here are represented in Fig. 4. Computations were performed for the identical initial approximation, equal to 0.2 W/(m·deg). Data of the simulation show that the main influence on the accuracy of solving the inverse problem being analyzed is exerted by the location of the first sensor, which should be in a zone relatively close to the surface being heated. The change in second sensor location here has weak influence on the accuracy of the inverse problem solution within sufficiently broad limits.

Then by using the algorithm elucidated in [6], planning of the temperature measurements was carried out. The same initial data as in the solution of the model inverse problem were utilized as a priori information here.

Solution of the extremal problem

$$\xi_0 = \arg\max\det[M(\xi, \lambda)] \tag{4}$$

was realized for the locally optimal measurement plan $\xi_0 = \{N, \bar{X}\}$, $\bar{X} = \{X_i\}_1^N$, where $M(\xi, \lambda)$ is the information matrix.

Computation of the information matrix was performed by using values of the response functions $\Theta_k(x, \tau) = \partial T(x, \tau)/\partial\lambda_k$ determined from the solutions of the boundary value problems obtained by differentiating the conditions of the direct heat conduction problem with respect to λ_k. The set of allowable plans Ξ was formed on the basis of results of the uniqueness theorem for the inverse problem under consideration [5] and was represented as follows:

$$\Xi = \{(N, \bar{X}): N \geqslant 1,\ 0 < x_i < b,\ i = \overline{1, N}\}.$$

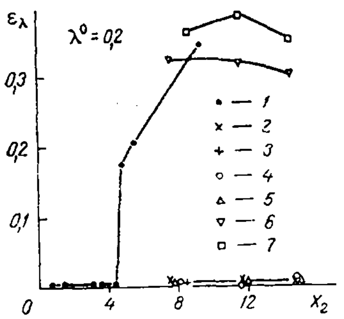

Fig. 4

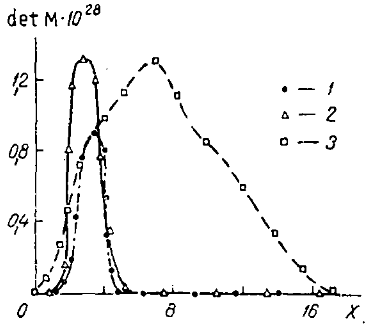

Fig. 5

Fig. 4. Dependence of the error ε_λ on the installation coordinates of one (1) or two thermal sensors: 2) $X_1 = 1.0$ mm; 3) 2.0; 4) 2.6; 5) 4.0; 6) 4.5; 7) 5.0. X_2, mm.

Fig. 5. Dependence of the planning criterion (det M) on thermal sensor installation coordinates: 1) N = 1; 2) N = 2 – 1 sect.; 3) N = 2 – 11 sect.

The search for the optimal plan (4) was constructed sequentially. Starting with N = 1 and increasing the amount of thermal sensors by 1, a selection of the optimal vector of the measurement point coordinates $\overline{X}$ was realized for each N from the condition

$$\overline{X} = \arg\max \det [M(\overline{X}, \lambda)], \; 0 < X_i < b, \; i = \overline{1, N}.$$

Results of solving the optimal location selection problem are represented in Fig. 5 for one and two thermal sensors, where the change in the planning criterion is illustrated as a function of the sensor installation coordinates. For two sensors, sections by planes drawn through the point of the maximal value of the criterion in parallel to the coordinate planes are shown here for the surface $\det M(X_1, X_2)$.

The most important outcome from the data obtained for the measurement planning is that one sensor should absolutely be installed in a sufficiently narrow domain near the heating boundary X = 0 to assure high reliability of the inverse problem solution in the experiment being analyzed. A further increase in the number of thermal sensors showed that two sensors are completely sufficient in this experiment since the placement of the second and successive sensors at one point is optimal for N > 2. The results obtained are in complete conformity with the data of computational experiments performed earlier.

Planning the measurements for experiment No. 2 results in analogous results. It is thereby proved that it is impossible to obtain reliable data about the heat conduction coefficient in the second experiment by using the described algorithmic support. This is a consequence of the absence of thermocouples in the X = 2.8 mm area in the second experiment.

The investigations performed show that application of computational experiments and optimal measurement planning is an inherent part of the complex procedure for the identification of heat transfer processes and permits approaching the question of reliability of the results being obtained validly.

NOTATION

P_0, T_0, M, stagnation pressure, stagnation temperature, and Mach criterion of the incoming stream; d, thermocouple distance from the surface being heated; T, the temperature; C, volume specific heat; λ, heat conduction; X, a coordinate; τ, time; N, number of thermal sensors; and b, specimen thickness.

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DETERMINATION OF THE SOURCE IN QUASILINEAR EQUATIONS OF THE PARABOLIC TYPE

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The uniqueness theorem is proven for the solution of the two-dimensional inverse problem for an unknown source function dependent on the solution of the direct problem and on the spatial coordinate.

We consider the heat equation in the region $D(T, x_0) = \{x_0 < x < \infty, 0 < t \leq T\}$

$$u_t = u_{xx} + q(u, x) + f(x, t) \tag{1}$$

subject to the boundary and initial conditions

$$u|_{t=0} = 0, \ x_0 \leqslant x < \infty, \tag{2}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_0} = 0, \ 0 \leqslant t \leqslant T, \tag{3}$$

plus the following condition on the solution at the point $x = x_0$:

$$u|_{x=x_0} = \psi(t, x_0), \ 0 \leqslant t \leqslant T. \tag{4}$$

We assume that the parameter x_0 could range from zero to infinity. The problem is to determine the function $q(u, x)$ for a given function $\psi(t, x_0)$.

THEOREM. Let the functions $f(x, t)$ and $\psi(t, x_0)$ satisfy the conditions $f(x, t) \in C[0, \infty) \times [0, T])$, $\psi(t, x_0) \in C^{1,0}([0, T] \times [0, \infty))$ and $\psi_t'(t, x_0) \geq \gamma$ for $t \in [0, T]$, where γ is a sufficiently large positive number. In addition, the consistency condition $\psi(0, x_0) = 0$ is assumed to be satisfied.

Then the solution $q(u, x)$ of the inverse problem is unique in the class of functions $q(u, x) \in C^{1,1}((-\infty, \infty) \times [0, \infty))$ satisfying the conditions

$$-q(0, x) \leqslant f(x, t) \leqslant \psi_t'(t, x_0) - q(\psi(t, x_0), x), \ x \in [x_0, \infty), \ t \in [0, T],$$

and for two arbitrary functions of this class $q_1(u, x)$ and $q_2(u, x)$ their difference $q(u, x) = q_1(u, x) - q_2(u, x)$ satisfies the inequality

$$\|q(u, x)\|_{W_\infty^{0,\alpha}([R_1, R_2] \times [0, \infty))} \leqslant c \|q(u, x)\|_{L_\infty([R_1, R_2] \times [0, \infty))}$$

for a certain value of $\alpha \in (0, 1)$.

Proof. The following maximum principle holds for the assumptions of the theorem.

Maximum Principle. If the conditions of the theorem are satisfied, then $u(x, t) \in C^{2,1}((0, \infty) \times (0, T]) \cap C([0, \infty) \times [0, T]) \cap L_\infty((0, \infty) \times (0, T])$ and the solution of the problem (1), (2), (4) satisfies the condition $0 \leq u(x, t) \leq \psi(t, x_0)$, $(x, t) \in D(T, x_0)$.

Furthermore suppose that there exist two solutions of the problem (1)-(4): $\{u_1(x, t), q_1(u_1, x)\}$ and $\{u_2(x, t), q_2(u_2, x)\}$. Then, putting $x = x_0$ in (1), we obtain the relation

$$q(\psi(t, x_0), x_0) = \psi_t'(t, x_0) - f(x_0, t) - u_{xx}|_{x=x_0}, \tag{5}$$

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